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Causality and the nature of information

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Abstract

Superluminal propagation of electromagnetic radiation and photon tunnelling have been studied by a variety of direct and indirect techniques. Especially the use of femtosecond terahertz pulses, whose electric field can be measured directly, has resulted in the (re)discovery of a number of effects in which light propagates over small distances faster than the speed of light in vacuum. Naturally, this brings up the question whether *information* can be exchanged superluminally. It has been shown in nearly all cases studied that the principle of causality applies to the underlying physical processes. It has been argued, however, that the principle of causality might have no bearing on the question of superluminal information transfer. It will be shown here that all the confusion stems from a vague definition of the concept of information and from ignoring noise. Once the concept of information (and noise) has been defined properly, it can be shown that if the principle of causality applies then useful superluminal information exchange is strictly prohibited.

1. Introduction

> *One of the problems has to do with the speed of light and the difficulties involved in trying to exceed it. You can't. Nothing travels faster than the speed of light with the possible exception of bad news, which obeys its own special laws [1].*

All sorts of "things" can travel with velocities exceeding the speed of light in vacuum without necessarily violating special relativity or causality. A problem only arises in the case of superluminal propagation of information. Many experiments have been performed this century that demonstrate superluminal propagation of signals [1] encoded on electromagnetic waves [2,3]. Superluminal effects generally occur when (electromagnetic) waves are forced through a structure or device in which the waves are evanescent. The wavevector of an evanescent field has one or more imaginary elements resulting in a purely exponential decay (or rise) of the field amplitude with distance in one or more spatial directions. The quantum-mechanical equivalent of evanescent-wave propagation is tunnelling in which the wave

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[1] A signal is any detectable waveform, which may contain information. Superluminal signal propagation does not necessarily imply superluminal transfer of information.

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function decays inside a classically forbidden barrier [2,4]. Superluminal propagation has been observed in frustrated total internal reflection, [5–10] phase-conjugating mirrors, [11] Bessel beams, [12,13] anomalous dispersion in absorbing and amplifying media, [14–19] dielectric mirrors and photonic band gaps, [20] diffraction, [21] and waveguides below the cut-off frequency [22–26]. It is sometimes argued that superluminal propagation of signals does not violate causality simply because the transmitted intensity is always lower than the intensity that would have been transmitted in vacuum [20,27,28]. This assertion is incorrect as superluminal propagation also occurs in amplifying media [29–32].

Understanding superluminal-propagation phenomena becomes ever more important as evanescent-waves are a crucial component of many new techniques and devices. For example, evanescent electromagnetic waves are central to many surface-sensitive spectroscopies and the various incarnations of near-field and tunnel microscopy [33]. Evanescent electron waves occur in sub-micrometer semiconductor devices [34]. The last few years there has been a huge increase in the development and use of terahertz pulses: Electromagnetic pulses typically containing one or only a few cycles of the field, with a centre frequency of about 1 THz, whose electric field can be measured directly using electro-optic sampling with a delayed femtosecond visible pulse [24–26]. Because the coherent detection of THz pulses allows the observation of temporal shifts much less than a pulse width, superluminal phenomena are more frequently observed, for example, in the Gouy phase shift [35,36] and pump-probe experiments [37].

Based on a series of experiments [22,23] on the propagation of microwaves through waveguides below cut-off, it has been argued that information may travel through a waveguide superluminally thereby, in effect, violating the principle of causality. In an infamous unpublished [38] experiment, Mozart's 40th symphony has been transmitted through a barrier at 4.7 c. Others have argued [2,39] that, as the waveguide obeys the (mathematical) principle of causality, information cannot, under any circumstance, be transmitted superluminally. The counter argument has been that, although the waveguide may obey the principle of causality, [40] "physically-realistic signals" are "bandwidth limited" and therefore the principle of causality does not apply. [38,40–42] These at times acrimonious discussions have not helped to clarify the issues. It appears that most of the confusion has been generated by an inadequate definition of "information". The concept of information has been very well defined in information theory [43] and can be linked to entropy in thermodynamics [44]. Using the principles of information theory, it will be shown in this paper that *useful* superluminal transfer of information is strictly impossible if the principle of causality applies. The main reason for this result is that information is spread out over space and time (i.e., it is not contained in a point) allowing for considerable fuzziness in the definition of arrival time. At the same time, (thermal) noise determines an interval of definition of information and it will be shown that information cannot escape from this interval. It will be found that these conclusions apply whether the signal pulse is bandwidth limited or not.

2. (Superluminal) propagation of light

The propagation of light through dispersive media has been described in detail in many places [45,46] and therefore a much-abbreviated version will be presented here. It will be assumed that all propagation phenomena relevant to the current discussion can be adequately described with plane waves propagating along the z-axis (Note that this assumption is not valid for frustrated total internal reflection). When a wave at frequency ω propagates through a certain length of material or some device, it will accumulate phase. If the Fourier transform of the field at the input is given by $\tilde{E}(0,\omega)$, the field at the output is given by

$$\tilde{E}(z,\omega) = \tilde{E}(0,\omega)\exp(\mathrm{i}\varphi(z,\omega)), \quad (1)$$

where φ is the accumulated phase. In the case of propagation through a dispersive medium, the accumulated phase is $\omega z n(\omega)/c$, where $n(\omega)$ is the frequency-dependent refractive index. In the case of propagation through a more complicated structure such as a waveguide (see below), it can be useful to define an effective refractive index. In order to understand what effect the accumulated phase has on the propagation of a short pulse, it is common to expand the phase as a Taylor series around the carrier frequency of the electromagnetic pulse as

$$\varphi(\omega) = \varphi_0 + \varphi_1(\omega - \omega_0) + \frac{1}{2}\varphi_2(\omega - \omega_0)^2 + \cdots, \quad (2)$$

where

$$\varphi_m = \partial^m \varphi(\omega)/\partial\omega^m|_{\omega=\omega_0}. \quad (3)$$

In this expansion, $\tau_\varphi = \varphi_0/\omega_0$ is the phase delay, $n_{\text{eff}} = \varphi_0 c/\omega_0 z$ is the effective refractive index, $\tau_g = \varphi_1$ is the group delay and φ_2 is the group-velocity dispersion [45]. The phase delay (and the corresponding phase velocity) is the delay experienced by the crests of the waves on traversing a certain distance. The group delay (and the corresponding group velocity) is the delay experienced by the envelope of the pulse. The group-velocity dispersion and all higher-order terms give rise to broadening of the envelope of the pulse.

In Appendix A, a detailed description is given of propagation through waveguides including expressions for the phase delay, group delay and group-velocity dispersion. The expression for the phase accumulated by an electromagnetic wave making a single pass through a waveguide of length L is (ignoring end effects)

$$\varphi(\omega) = \omega_c L c^{-1}\sqrt{(\omega/\omega_c)^2 - 1}. \quad (4)$$

It can be seen from this expression that at frequencies below the critical frequency ω_c, the accumulated phase is purely imaginary, implying that both the phase delay and group delay are imaginary. As the real part of these delays is zero, this means that the pulse propagates through the waveguide with zero delay (while suffering considerable attenuation). The group-velocity dispersion is also imaginary and small, which means that the pulse narrows slightly while traversing the waveguide. These conclusions still hold true when multiple reflections inside the waveguide and end effects are taken into consideration. However, two new features are introduced: The transmission at zero frequency becomes zero and the effective refractive index becomes negative.

Fig. 1 shows calculated transmission and effective refractive-index spectra for a waveguide with the effect of multiple reflections included. As the effective refractive index below the cut-off frequency is negative, this implies that electromagnetic waves emerge from the waveguide before entering it. Interestingly, this effect is entirely due to the phase shifts incurred by reflection off the

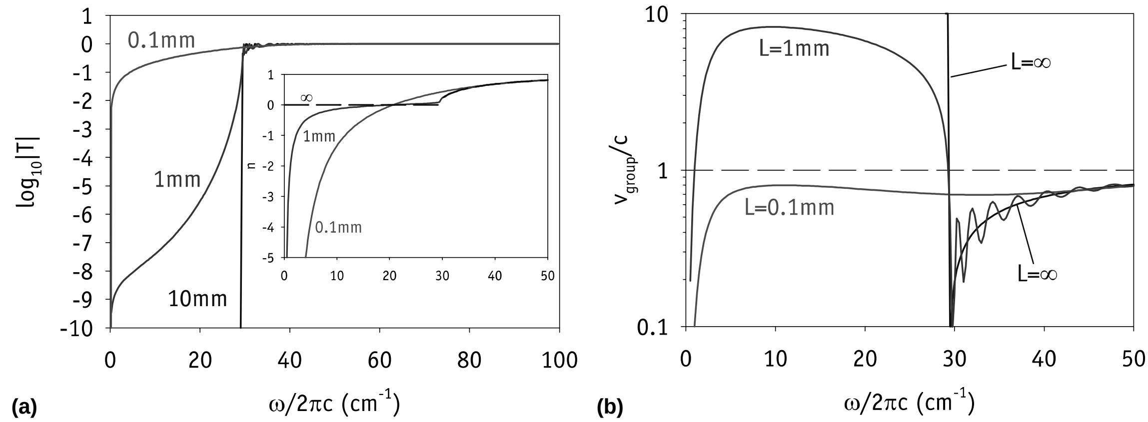

Fig. 1. Theoretical calculation of propagation of the lowest-order TE mode through a cylindrical metal waveguide with a diameter of 200 μm (cut-off frequency 0.88 THz or 29 cm^{-1}) using Eq. (28). (a) Field-transmission spectra for waveguides with lengths $L = 0.1$, 1, and 10 mm with multiple reflections inside the waveguide included. Note that the transmission is zero at zero frequency. The inset shows the effective refractive index for the same waveguides. (b) Group velocity calculated from the effective refractive index in (a).

boundary between vacuum and the waveguide. This may seem like an "unfair" way of advancing a wave but it can nonetheless result in the observation of the peak of a pulse emerging from a waveguide before entering it [24]. Similar temporal advances due to phase shifts at a boundary have been seen in recent pump-probe studies [37]. It can be seen in Fig. 1 that the group velocity exceeds the vacuum speed of light for sufficiently long waveguides. When the length of the waveguide is on the same order as the diameter, edge effects make the group-velocity subluminal. In the longer waveguides, one can observe oscillations in the transmission due to Fabry–Perot effects of the non-evanescent waves.

3. Causality

The calculations presented here and many experiments [22–24] have established that electromagnetic waves can propagate superluminally through a waveguide below the cut-off frequency. This is not necessarily in conflict with special relativity theory. Only if *information* were to travel with superluminal velocity would there be an inconsistency with special relativity. The question that has to be answered is therefore, with what speed does information travel through the waveguide? Clearly, this is not the phase velocity. The phase velocity is the propagation velocity of the crests of the electromagnetic wave or equivalently, the velocity of the monochromatic components of a pulse. No information can be encoded in a monochromatic wave or, to be more precise, the information encoded in a monochromatic wave is entirely delocalised over time and space. Therefore, the negative effective refractive index shown in Fig. 1 has no direct bearing on the signal velocity and does not indicate a non-causal effect. Fig. 1 also shows that the group velocity can exceed the speed of light, the effect being most pronounced for long waveguides where end effects are minimal. If the group velocity were equal to the information velocity, this would suggest a violation of some form of causality. In this respect, it is interesting to consider the velocity with which electromagnetic energy travels through the waveguide. If an electromagnetic pulse at the input of a dispersive medium has instantaneous intensity $I(t) \propto |E(t)|^2$ then at the output, it will have an intensity proportional to $I(t-\tau_g)$ if the group-velocity dispersion and higher-order terms are ignored. The average arrival time of the energy is determined by the centroid delay as

$$\tau_c = \int_{-\infty}^{\infty} dt\ tI(t,z) \Big/ \int_{-\infty}^{\infty} dt I(t,z). \tag{5}$$

This expression is valid even if the slowly varying amplitude approximation [47] is not. Thus, at the exit of the dispersive medium (assuming $I(t)$ to be normalised)

$$\tau_c = \int_{-\infty}^{\infty} dt\ tI(t-\tau_g) = \int_{-\infty}^{\infty} dt\ tI(t) + \tau_g. \tag{6}$$

As the first term in Eq. (6) is a constant offset, this shows that the centroid-delay difference (free space vs. dispersive medium) equals the group-delay difference in the approximation that the group-velocity and higher-order dispersion terms can be ignored. In the calculations that have been performed (see below), this approximation appears to be quite good. Thus, one can conclude that the energy in the pulse can also travel superluminally through the waveguide. This might *suggest* that causality may be violated. In an experiment using beams of correlated photons in which one beam travels through a photonic band gap, it was shown that single-photon wavepackets can also travel with superluminal speed [48].

The above results are all very worrying. Is it possible to transmit information superluminally and hence, in some inertial frames, to receive a signal before it has been sent? Perhaps the answer lies in an examination of the principle of causality [16]. If the response of a system with length z is causally related to the input to the system, the output electric field must be related to the input field by

$$E_{out}(z,t) = \int_{-\infty}^{\infty} d\tau E_{in}(0,t-\tau) r(\tau - z/c), \tag{7}$$

where the response function $r(t-z/c)$ is zero for $t < z/c$. Any value of the response function at times $t < z/c$ would violate causality in some Lorentz frames (a transmitter receiving its own message before sending it). In the case of a waveguide, the time-response function is found from the Fourier transform of the transmission function Eq. (28). The effective index of the waveguide tends to unity as the absolute value of the complex frequency tends to infinity. Therefore, the Fourier transform can be performed by contour integration by closing the contour in the upper half of the complex-frequency plane if $t - z/c < 0$. Therefore, the waveguide response is causal if there are no poles in the upper-half plane of the integrand [46,49]. It has not been possible to find an analytical expression for the position of the poles but a numerical search only resulted in an infinite series of poles in the lower half of the complex-frequency plane. As a further causality proof, the Fourier transform of Eq. (28) has been performed numerically. The first response of the system is a Dirac delta function arriving at $t = z/c$. Thus, electromagnetic-wave transmission through a waveguide is causal even when one includes the effect of multiple reflections of evanescent waves each travelling at infinite speed. Interestingly, it is assumed that the principle of causality Eq. (7) applies to all physically realistic systems although there is no proof that this will always be the case.

The principle of causality has been used previously [16,39,44] as an argument to define information as a point of non-analyticity. From that point of view, the principle of causality is proof that superluminal communication is impossible. In some cases, the response function has such a form that it appears as if information is travelling superluminally. However, the tunnelling device in effect performs an extrapolation into the future [2,31,50]. This has been demonstrated beautifully in experiments using resonant amplifiers in which the peak of the output pulse may leave the circuit before the peak of the input pulse arrives at the input port [16,51] without violating causality [16]. Such an extrapolation could conceivably be performed even for a pulse travelling through free space. Taylor expansion of the wing of a pulse arriving before the peak could be used to predict the arrival time of the peak. Thus, genuinely new information is contained only in points of non-analyticity (the "front") and these travel with the front-velocity, which is equal to the speed of light in vacuum [39,46]. This can be understood as arising from the fact that discontinuities have Fourier components at infinite frequency and in a waveguide these infinite frequency components would be above cut-off.

Unfortunately, the above argument to define information as a point of non-analyticity is weak. For example, consider extrapolation into the future by Taylor expansion. Fig. 2 shows a Gaussian

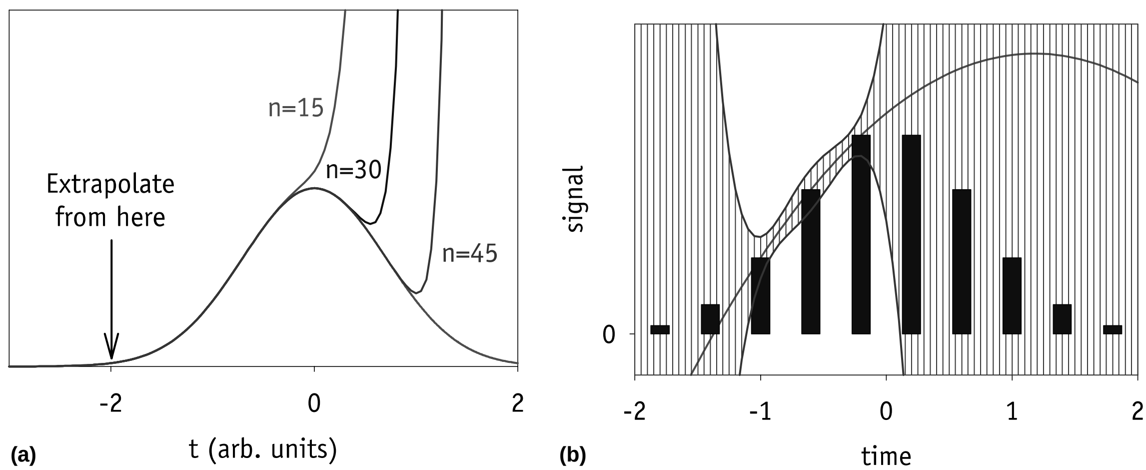

Fig. 2. (a)Taylor expansion of the Gaussian function $\exp(-t^2)$ around $t = -2$ up to 15th, 30th and 45th order. (b) Gaussian pulse approximated by a parabolic curve using the Lagrange formula [52] on samples taken at $t = -1, -0.6$, and -0.2. The hatched area shows the uncertainty in the parabolic extrapolation that is caused by noise in the three samples of the signal.

pulse peaking at $t = 0$ and three Taylor expansions around $t = -2$ of 15th, 30th and 45th order. It can be seen that in order to extrapolate about two pulse widths into the future and predict the arrival of the peak, a 30th-order Taylor expansion is required in this case. The detector at the receiving end of the communication system will have to take 31 samples of the waveform around $t = -2$ in order to perform this 30th-order Taylor expansion. Each of these samples must contain a finite number of photons (in fact rather more than one photon) for this procedure to work. In addition, the sampling will have to be performed quickly with respect to the pulse width but slowly enough so that enough photons are detected to obtain meaningful samples. No matter how carefully the sampling and extrapolation is performed, the procedure will always result in a finite probability of making an error in predicting the future. This error can be made arbitrarily small by using a signal pulse with a peak amplitude that approaches infinity. However, to extrapolate a significant amount of time into the future (for example, ten pulse widths) may require an unfeasibly large number of photons in the signal pulse. Extrapolation into the future only works for analytic and noiseless signals. The thermal and shot noise inherent in the production and detection of electromagnetic waves [44] prohibits extrapolation into the future by "significant" amounts.

4. Information

It has been established that extrapolation into the future is of limited value and therefore one may wonder whether there is a pressing reason to define information as a point of non-analyticity. Is a point of non-analyticity in fact physically realistic? It is useful in this context, to consider the definition of information in information theory [43]. In a practical digital communication system, bits will be transmitted using bell-shaped laser pulses. Laser pulses with energy above some cut-off level will be considered to represent a one-bit, ones with energy below the cut-off as a zero-bit. However, because the pulses represent an analogue continuous signal, this does not necessarily mean that these pulses contain an amount of information exactly equal to one bit. Consider a laser pulse with a constant level of Gaussian noise. (At the most fundamental level, every degree of freedom has thermal noise with an average energy proportional to $k_B T$ [44]. Based on the central limit theorem, [53] it will be assumed here that thermal noise is a Gaussian stochastic process. See below.)

To represent a continuous signal faithfully, it has to be sampled at different points in time. According to the sampling theorem, [54] in order to obtain a faithful representation of the signal it has to be sampled at a rate twice its bandwidth $2W$. The problem of defining the bandwidth in this context will be discussed in a moment. The required accuracy of each sample is determined by the amount of noise present. Thus, if the signal strength (for example, the electric field strength) is e_s and the standard deviation of the noise is e_n, the total rate of information transfer is given by [54] $2W \log_2 (e_s/e_n)$ expressed in units of bits per second. Expressed in terms of signal power p_s and noise power p_n (variance), this is $W \log_2 (p_s/p_n)$.

What is the bandwidth of a continuous signal? Consider a Gaussian pulse in the time domain and its Fourier transform (see Fig. 3), which is again a Gaussian in the frequency domain. It could be argued that such a pulse has an infinite bandwidth as the Gaussian spectrum falls off exponentially towards infinite frequency. However, this is not a useful definition of bandwidth since the noise power spectrum also depends on the sampling rate. For example, in the case of Gaussian noise, the noise power is related to the sampling rate by $p_n(W) \propto \sqrt{W}$. Therefore, in the case of a practical digital-communication system using clock-shaped laser pulses representing 0's and 1's, the detector sampling rate will be set approximately equal to the bit rate. For this choice of sampling rate, the signal-to-noise ratio is maximised, the bit-error rate minimised and the detected information content is one bit per pulse.

A result of the above definition of the rate of information transfer is that a continuous signal containing a point of non-analyticity requires *infinite channel capacity* to be transmitted faithfully. The reason is that to define a discontinuity in the time domain requires a well-defined spectrum to

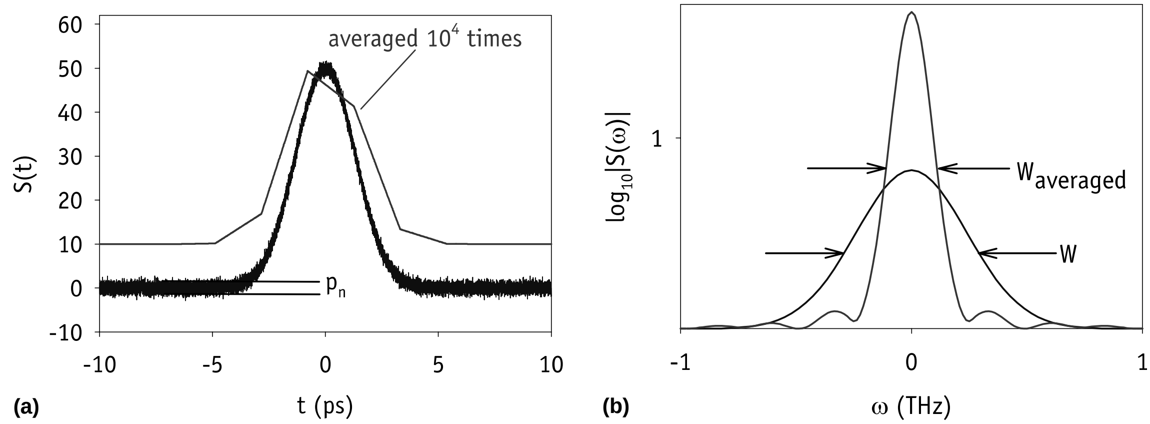

Fig. 3. (a) A Gaussian pulse, $50\exp(-\Omega^2 t^2/4\log 2)$ with $\Omega = 2\pi/5$ THz, sampled every 2 fs with a Gaussian noise component with $p_n = 1$. Also shown are the same data averaged 10 240 times and shifted up by 10 units for clarity. (b) Fourier transform of the pulses shown in (a). W is the bandwidth of the original signal and W_{averaged} that of the averaged signal. The bandwidth of the averaged signal is reduced by about a factor of two. However, the noise level (as seen on a logarithmic scale) is reduced by about 100 times in the averaged signal. Therefore, an optimum signal-to-noise ratio is achieved when the signal is sampled at a rate equal to the rate at which pulses are sent.

infinite frequency ($W \to \infty$). Of course, nearly all analytic pulse shapes require infinite channel capacity. However, in the real world, signals are associated with a finite amount of noise limiting the required sampling bandwidth. Thus, any definition of information has to take into account this noise. As a result, an investigation into whether information can travel superluminally also has to take into account noise. Defining information in terms of analytical waveforms or points of non-analyticity is physically unrealistic.

5. Points of non-analyticity

Before continuing, the properties of points of non-analyticity will be considered in some more detail. Points of non-analyticity come in various flavours with different properties, ordered as: The Dirac delta function, the Heaviside step function, a step in the first derivative, second derivative, etc. The functions in this list are related by integrals. The Heaviside step function is

$$\theta(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau, \tag{8}$$

the integral of the Heaviside step function is a step in the derivative, etc. To find the Fourier transforms of these discontinuities, the convolution theorem can be used to find

$$F\left(\int_{-\infty}^{t} f(\tau)\,d\tau\right) = \tilde{f}(\omega)[\pi\delta(\omega) - i\omega^{-1}]. \tag{9}$$

The Heaviside step function (or any of its integrals) is not a realistic signal as it contains infinite energy. Any realistic signal has to fall off to zero for $|t| \to \infty$, which will modify the Fourier transform of Eq. (9) for frequencies close to $\omega = 0$. Eq. (9) does show, however, that the spectrum of any discontinuity will fall off as ω^{-n} for $|\omega| \to \infty$, where n is the order of the discontinuity ($n = 0$ for the Dirac delta function, $n = 1$ for the Heaviside step function, etc.). The properties of discontinuities lead to a (well known) problem. Consider, for example, a single-sided exponential pulse with carrier frequency ω_0, which has finite energy.

$$E(t) = \theta(t)\exp(-i\omega_0 t - \gamma t). \tag{10}$$

Its Fourier transform is given by

$$\tilde{E}(\omega) = \int E(t)e^{+i\omega t}\,dt = (-i[\omega_0 - \omega] - \gamma)^{-1}. \tag{11}$$

One would like to calculate the width of the pulse in the time domain and the width of its spectrum. There are many equally valid definitions of width [44] but here the statistical variance or twice the standard deviation of the intensity-distribution function of the electromagnetic field will be used as the definition of width. Thus, the width of the single-sided exponential pulse is

$$\langle t^2 \rangle = \int |E(t)|^2 t^2 \, \mathrm{d}t = (4\gamma^3)^{-1}, \quad (12)$$

which is finite but its spectral width

$$\langle \omega^2 \rangle = \int |\tilde{E}(\omega)|^2 \omega^2 \, \mathrm{d}\omega \quad (13)$$

is infinite. An infinite spectral width is nothing to be particularly worried about; it simply means that the spectrum of the pulse is delocalised. Even if the spectral width is finite, the spectrum could still extend to infinite frequency (which is the case, for example, with a Gaussian pulse). The problem of infinite spectral width will disappear if one chooses a signal pulse that only has a discontinuity in any of its derivatives. For example, [16]

$$E(t) = \theta\left(\frac{1}{2}\tau - |t|\right) \exp(-\mathrm{i}\omega_0 t) \cos(\pi t/\tau), \quad (14)$$

which is clock shaped within the interval $(-\tau/2, \tau/2)$, has discontinuities in its first derivative at $t = \pm\tau$. The power spectrum of this pulse therefore falls off as ω^{-4} and its spectral width $\langle \omega^2 \rangle$ is finite. Of course, the higher moments of the spectral distribution are still infinite.

There is no law of physics stating that all the moments of a (physically realistic) probability distribution function have to be finite. The Schrödinger equation, Newton's equation and the Maxwell equations have valid solutions containing points of non-analyticity in the time domain or the frequency domain. This does not necessarily imply that points of non-analyticity do in fact occur in nature. Points of non-analyticity give rise to moments of the conjugate distribution function with infinite value. This is the reason that an infinite channel capacity is required to transmit them.

In a number of papers, [38,41,42] it has been argued that information can be transmitted superluminally using "bandwidth-limited signals," that is, signals that have their entire spectrum below the cut-off of a waveguide. To investigate the physical reality of such a spectrum, one may consider a pulse spectrum (a) with an amplitude tending to zero as the cut-off is approached, (b) with an amplitude that is zero above the cut-off and (c) without discontinuities in the amplitude at the cut-off. An example of such a spectrum is

$$\tilde{E}(\omega) = \theta\left(\frac{1}{2}\Omega - |\omega|\right) \cos(\pi\omega/\Omega), \quad (15)$$

where Ω is the cut-off frequency of the signal spectrum, which could be chosen such that it is below the cut-off frequency of the waveguide. Its Fourier transform is given by

$$E(t) = \frac{1}{2\pi} \int \tilde{E}(\omega) \mathrm{e}^{-\mathrm{i}\omega t} \, \mathrm{d}\omega = \frac{\Omega \cos(\frac{1}{2}\Omega t)}{\pi^2 - \Omega^2 t^2}. \quad (16)$$

One can again calculate the width of the pulse and its spectrum using the definition in terms of the statistical variance. For the cos-pulse, these variances are given by

$$\langle t^2 \rangle = \int |E(t)|^2 t^2 \, \mathrm{d}t = \pi^2/\Omega^2,$$
$$\langle \omega^2 \rangle = \int |\tilde{E}(\omega)|^2 \omega^2 \, \mathrm{d}\omega = \frac{(\pi^2 - 6)}{12\pi^2} \Omega^2. \quad (17)$$

It can be seen that both the temporal width and the spectral width are finite for this type of pulse. However, the higher-order moments of the temporal distribution function are infinite implying that the cos-pulse is delocalised in the time domain. Therefore, in order to produce a strictly bandwidth-limited pulse such as that in Eq. (15), the temporal signal has to be controlled from $t = -\infty$ to $t = +\infty$. This is impossible, if only because of the finite age of the universe. More practically, the experimenter presumably switches the equipment off at the end of a (long) day, which will limit the duration over which the pulse can be defined and will broaden the spectrum. Therefore, inevitably, the pulse spectrum will bleed into the region above the cut-off frequency. A bandwidth-limited spectrum is as physically unrealistic as a point of non-analyticity in the time domain.

6. What is information and can it go faster than the speed of light or what?

At first sight, it might appear that the concept of information is rather arbitrary and depends on an agreement on what constitutes information. However, independent of this arbitrary agreement, every communication system based on electromagnetic waves must use a receiver detecting photons. One "bit" of information is received when the detector has received a sufficient number of photons to be sufficiently sure that an on-bit rather than an off-bit was received. In other words, information may be defined as a pulse (or some other shape) that a detector can distinguish from noise. This does not mean that information is defined simply by the peak of a pulse: The detector should be able to distinguish between on and off bits with an acceptable signal-to-noise ratio, which requires a finite amount of averaging time and a finite number ($\geqslant 1$) of photons. The detector has only one shot at detecting a bit and cannot perform an ensemble average to improve the signal-to-noise ratio. This implies that any communication system has a finite bit-error rate. It also implies that any signals below the noise level of the communication system might as well not exist. To investigate superluminal propagation of information (or lack thereof), one may consider any pulse shape as long as a noise model is incorporated. When noise is included, it will be found that even signals that have infinite moments (in the time or frequency domain) can be described without problems.

Appendix A describes in detail how propagation of electromagnetic pulses through a waveguide has been calculated. These calculations include the effects of phase shifts occurring at the boundaries and multiple reflections inside the waveguide. Fig. 4 shows some of the simulation results. For example, Fig. 4(a) shows the time-dependent intensity of Gaussian pulses propagated through waveguides with increasing lengths replacing equivalent amounts of free space. It can be seen that the peak of the Gaussian pulse propagates through the waveguide with nearly infinite speed. The centroid delay (average arrival time of the energy) also indicates a nearly infinite speed. The field and its Fourier transform are given by

$$E(t) = \exp\{-t^2/2\sigma^2\},$$
$$\tilde{E}(\omega) = \sqrt{2\pi\sigma^2}\exp\left(-\frac{1}{2}\omega^2\sigma^2\right), \tag{18}$$

which have variances $\langle t^2\rangle = \sigma^2/2$ and $\langle\omega^2\rangle = 1/(2\sigma^2)$. One might arbitrarily [44] define the pulse width by twice the standard deviation $\Delta t = 2\langle t^2\rangle^{1/2} = 2^{1/2}\sigma$ and the spectral width by twice the spectral standard deviation $\Delta\omega = 2\langle\omega^2\rangle^{1/2} = 2^{1/2}/\sigma$. Note that the Heisenberg energy-time uncertainty relation ($\Delta E \cdot \Delta t \geqslant \frac{1}{2}\hbar$) relates standard deviations [55]. For the longer waveguides, the advance of the pulse (as measured by the centroid delay) is more than a pulse width (as defined above). This superluminal advance comes at a price: The amplitude of the transmitted wave is strongly attenuated. A Gaussian pulse has a continuous spectrum extending to infinite frequency. Part of the spectrum will therefore extend above the cut-off frequency of the waveguide. Since the non-evanescent waves are not attenuated, for very long waveguides the transmitted pulse will be dominated by non-evanescent components. Fig. 4(b) shows the time-dependent intensity of a Gaussian pulse propagated through a waveguide with such a length that the non-evanescent component of the pulse begins to dominate the evanescent component. In that case, the pulse develops a trailing component. Fig. 4(c) shows the propagation of a cos-pulse (Eq. 16) with a pulse width chosen such that its entire spectrum is below the cut-off frequency of the waveguide. In that case, the entire pulse is evanescent and could conceivably be propagated through waveguides of arbitrary length. Again, it can be seen from the calculated centroid delays that the energy in the pulse travels through the waveguide at infinite speed.

The simulations shown in Fig. 4 are of analytical functions and do not include noise. Fig. 5 shows a simulation of a Gaussian pulse with noise. The noise has been added to the field, has a Gaussian distribution with standard deviation σ (in the field), and is uncorrelated (white spectrum). The simulation shows that adding noise has little effect except that, as the transmitted pulse is attenuated, the signal-to-noise ratio in the transmitted pulse becomes smaller as the waveguide becomes longer. According to the definition of information as $\log_2(p_s/p_n)$, attenuation of the signal implies loss of information. If the waveguide is chosen long enough, the attenuation will be so

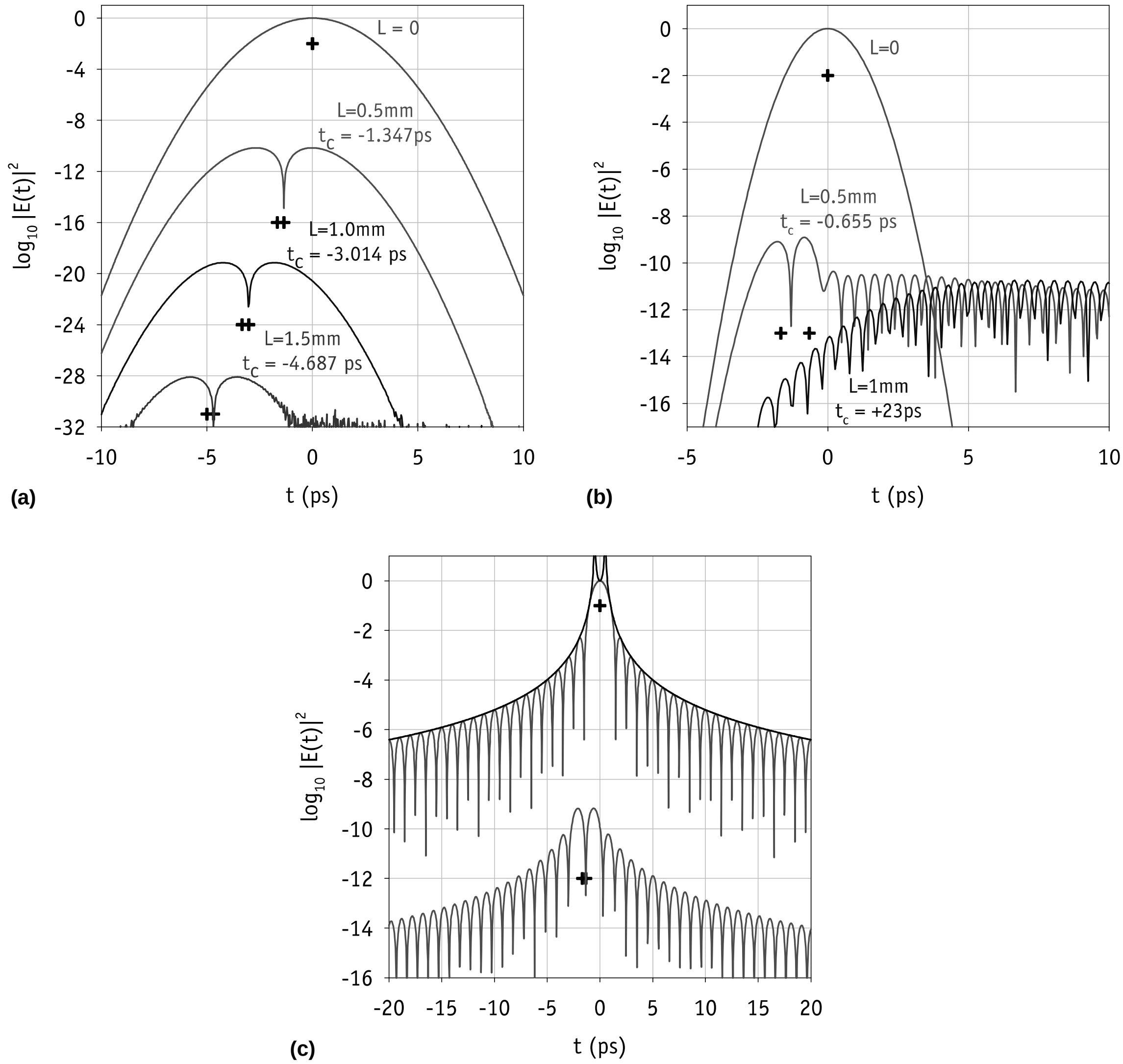

Fig. 4. Simulated propagation of two types of pulses through waveguides of various lengths. (a) Propagation of a Gaussian pulse $\exp(-(t/2ps)^2)$ with width $\Delta t = 2$ ps through waveguides of length $L = 0$, 0.5, 1.0 and 1.5 mm and diameter $d = 0.175$ mm (cut-off frequency 1 THz). The crosses indicate the expected arrival time of the pulse if it had travelled at infinite speed through the waveguide and the centroid delay time calculated from the simulated pulse shapes. (b) Same as (a) except for shorter pulse width $\Delta t = 1$ ps. (c) Propagation of a cos-pulse (Eq. 16) with $\Omega/2\pi = 1$ THz and pulse width $\Delta t = 1$ ps through waveguides with $L = 0$ and 0.5 mm, and $d = 0.175$ mm.

severe that the peak of the pulse becomes lower than the noise level. At that point, all information is lost. Therefore, if one would like to achieve a certain superluminal advance (by choosing a waveguide with a certain length), one has to choose the energy (and therefore peak amplitude) of the signal pulse such that after traversal of the waveguide, there is sufficient energy left for the detector to detect the signal with sufficient signal-to-noise. In the example shown in Fig. 5, the noise level has been set to $\sigma_{\text{field}} = 10^{-6}$. This means that if the peak amplitude of the input pulse is unity, the pulse can suffer an attenuation of 10^6 in the waveguide before all information is lost. However,

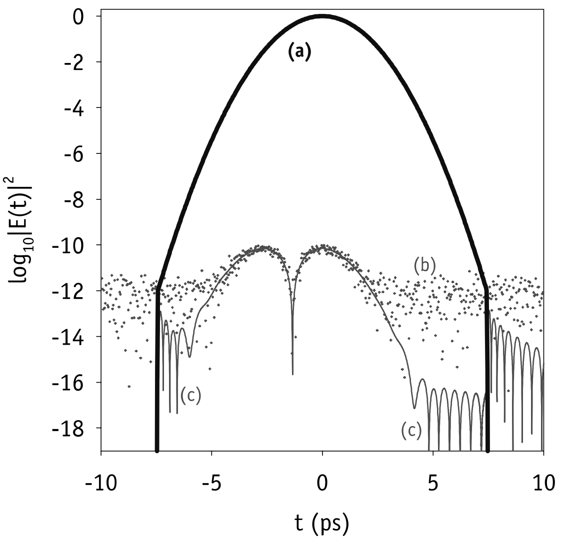

Fig. 5. Computer simulation of a Gaussian pulse $\exp(-(t/2ps)^2)$ travelling through a waveguide. The spatial z-coordinate runs from −20 to 20 mm and has been divided into a number of steps. The simulation time coordinate runs from $-\Delta z/c$ to $+\Delta z/c(\Delta z = 40$ mm, hence, $\Delta t = \pm 133.4$ ps) and has been divided into 8192 steps for (a) and (b) ($\delta t = 16.2$ fs) and 65536 steps for (c) ($\delta t = 2.0$ fs). The waveguide starts at $x = 0$, has a length of 0.5 mm and a diameter of 175 μm (cut-off frequency 1 THz). (a) Gaussian pulse chopped off at the 10^{-6} level and propagated through air. (b) Gaussian pulse with Gaussian noise with $\sigma = 10^{-6}$ propagated through the waveguide. (c) Pulse shown in (a) propagated through the waveguide.

this also implies that the input pulse must be well defined over a large time interval. The Gaussian input pulse in Fig. 5 has an intensity greater than 10^{-6} in the time interval $t = \pm 7.5$ ps. The superluminal advance made by propagating through the waveguide is only 1.3 ps (as measured by the centroid delay), which is much smaller than the interval over which the pulse was defined. As can be seen in Fig. 4(c), this effect is even more pronounced if the Gaussian pulse is replaced by a cos-pulse. In Fig. 4(c), the superluminal advance is 1.33 ps with a power loss of 6.67×10^{-10}. Defining the input cos-pulse down to the 6.67×10^{-10} level, requires a time interval of $t = \pm 98$ ps.

In all the examples discussed here, the superluminal advance is smaller than the time interval over which the input pulse has to be defined. One might say that in these examples superluminal exchange of information does take place. However, it is not a *useful* form of superluminal information exchange. It is equally valid to say that in these examples no superluminal exchange of information has taken place because the information is spread out over a time interval that is larger than the advance. Worse yet, superluminal propagation is seen to be accompanied by loss of information. *The only situation in which one might say that useful superluminal information exchange has taken place is if the superluminal advance is larger than the interval over which the input pulse is defined.*

Fig. 5 also shows a simulation of a Gaussian input pulse chopped off at the 10^{-6} level, introducing two points of non-analyticity in the field amplitude. As expected, [46] these points of non-analyticity are transmitted by the waveguide at exactly the speed of light in vacuum because the waveguide response is causal. Since the waveguide is not an amplifier (and cannot be at all frequencies [49]), the amplitudes of the transmitted discontinuities are the same as those at the input. Therefore, the transmitted chopped-off pulse reproduces exactly the transmitted noisy pulse wherever the field strength is larger than the noise level σ. This is not surprising: It is meaningless to speak of the "shape of the pulse below the noise level." Information is only present if it can be measured.

This can be expressed mathematically and generalised to any causal system. If $S_{\rm in}(t)$ is a signal pulse that does not include noise then if this pulse is transmitted by a causal system, the output signal is given by

$$S_{\rm out}(t) = \int_{z/c}^{\infty} d\tau \; S_{\rm in}(t-\tau) r(\tau - z/c). \tag{19}$$

In this case, it is difficult (if not impossible) to determine whether useful (as defined earlier) superluminal signal exchange is possible or not. However, any physically realistic signalling device introduces thermal noise [44]. If the signalling device is a blackbody radiator emitting linearly polarised radiation, [2] it can be shown [56] that the

[2] It is possible to generalise to unpolarised blackbody radiation but this would be tedious and add little to the argument.

electric field amplitude is a Gaussian random process with variance $\sigma^2 \propto \hbar\omega[\exp(\hbar\omega/k_B T) - 1]^{-1}$. Therefore, the output signal of a physically realistic system is

$$S_{\text{out}}(t) + N_{\text{out}}(t) = \int_{z/c}^{\infty} d\tau \, [S_{\text{in}}(t-\tau) + N_{\text{in}}(t-\tau;\sigma)] r(\tau - z/c), \quad (20)$$

where $N_{\text{in}}(t;\sigma)$ is the Gaussian random process. As pointed out above, the signal detector cannot perform an ensemble average and therefore cannot recover $S_{\text{out}}(t)$ without noise. Thus, from the point of view of information content, this expression is identical to the expression

$$S_{\text{out}}^{\text{chopped}}(t) + N_{\text{out}}(t) = \int_{z/c}^{\infty} d\tau \, [\hat{f}(\sigma) S_{\text{in}}(t-\tau) + N_{\text{in}}(t-\tau;\sigma)] r(\tau - z/c), \quad (21)$$

where $\hat{f}(\sigma)$ is an operator returning zero if the function following it is below the value σ. From Eq. (21), it can be seen that the output signal can never have a value outside the interval defined by the two points of non-analyticity introduced by the $\hat{f}(\sigma)$ operator. $S_{\text{out}}(t)$ only differs from $S_{\text{out}}^{\text{chopped}}(t)$ when $S_{\text{out}}(t)$ is below the noise level and therefore fundamentally unmeasurable. "Unmeasurable" here means that the bit-error-rate is 50%, i.e., the receiver randomly registers zero or one bits. This proves that *useful* superluminal exchange of information is strictly impossible. The form of the response function $r(t)$ does not enter into these considerations and therefore the conclusions are general. They also apply if the system amplifies [29–32] the signal rather than attenuating it. For the same reason it is not possible to obtain superluminal transfer of information by filtering out non-evanescent components of the signal if the filter is causal. Superluminal propagation of information is only possible by distorting the signal within the interval of definition. Thus, the maximum possible temporal advance is equal to the amount of time spent by the transmitter producing the signal in the first place. It is for this reason that causal paradoxes are avoided.

7. Conclusion

In the proof in the previous section, fronts where only introduced as a mathematical device. No transmitter has in fact to produce a front nor does any channel have to transmit a front. The mathematical front, buried in the noise and therefore fundamentally unmeasurable, is only used to prove that (a) information is spread out over a certain temporal interval of definition and (b) information cannot escape from this interval. In addition, it was found that the information content of a signal tends to reduce (or at most stay the same) during superluminal propagation, which is in essence the second law of thermodynamics [44]. As a result, useful superluminal transfer of information is strictly prohibited.

Up to this point, only information contained in a single pulse has been considered but an extension to pulse trains is obvious. The only important difference is that non-evanescent components of the signal are much more important in the case of pulse trains. Any non-evanescent component will travel with at most the speed of light in vacuum, will not be attenuated, and may therefore cause a reduction of the signal-to-noise ratio. In all experimental "demonstrations" of superluminal communication so far, [42] the temporal advance made has always been less than the inverse bandwidth of the signal. A back-of-the-envelope calculation shows [25] that as the superluminal advance becomes larger than the inverse bandwidth of the signal, non-evanescent components of the signal begin to dominate the evanescent ones resulting in a dramatic reduction of the signal-to-noise ratio for detecting pulse trains.

It has been argued previously [38,40–42] that, because the energy of a signal pulse is finite, any realistic signal must be bandwidth limited and therefore that it should be possible to construct a signal pulse that is entirely evanescent. Such a signal pulse should then result in the superluminal exchange of information and causal paradoxes. However, a strictly bandwidth limited pulse, e.g., a pulse that only has frequency components below the critical frequency of a waveguide, has an infinite extend in the time domain. This means that the information in this pulse is also delocalised over time. As the signalling system has to be switched on and off, the signal pulse will inevitably become localised in time even if the switching process is gradual. In addition, the energy of the signal pulse may be finite on average; however, it will fluctuate from shot to shot. This means there is a finite (although extremely small) probability for any signalling device to emit, for example, a gamma-ray photon. Frequency limitation is therefore not "a fundamental property of physical signals."

This paper has concentrated on the properties of information contained in evanescent electromagnetic waves. One may speculate that the same arguments may be applied to quantum mechanics, i.e., the wavefunctions of particles with non-zero rest mass. One might consider the notion that the only particles of interest are those that will be measured at some point in time. It seems that a measurement automatically implies the interaction with a thermal heat bath, which will result in the loss of information about that particle and an increase in the entropy [44].

In this paper, only linear optics has been considered in detail. However, it can be shown [57] that the principle of causality can be generalised to non-linear optics. This is not surprising as the Heisenberg equation of motion for a time-dependent operator has (local) causality effectively build in. Therefore, all the considerations regarding superluminal information transfer also apply to nonlinear optics.

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Appendix A. Waveguide Propagation

The waveguide-propagation equations for an unended cylindrical waveguide have been derived in detail in many other places. The derivation relies on the assumption that the waveguide walls are perfectly conducting in order that the field vanishes on the surface. The propagating fields are then identified by the number of nodes in the radial and the tangential direction. When waves propagate over a certain distance through the waveguide, they will accumulate phase, which for TE-modes is given by [58]

$$\varphi(\omega) = \omega_c L c^{-1}\sqrt{(\omega/\omega_c)^2 - 1}, \omega_c = 2xcd^{-1}, \quad (22)$$

where ω_c is the cut-off frequency, d is the waveguide diameter and L its length. For the lowest order TE_{11} mode, the parameter x has the value 1.841 [58]. All other modes have a cut-off frequency higher than the TE_{11} mode. Above the cut-off frequency, the transmission is unity (φ is real) and the waveguide is dispersive. Below cut-off, transmission is less than unity (φ is imaginary) and the accumulated (real) phase is zero. The (effective) refractive index derived from Eq. (22) is

$$n(\omega) = \sqrt{1 - \omega_c^2/\omega^2}. \quad (23)$$

As the phase delay is Re φ/c, the phase velocity tends to infinity as cut-off is approached. The group delay derived from Eq. (22) is

$$\begin{aligned}\tau_{group} &= \partial\varphi(\omega)/\partial\omega = \omega L c^{-1}(\omega^2 - \omega_c^2)^{-\frac{1}{2}} \\ &= \frac{L^2}{c^2}\frac{\omega}{\varphi(\omega)}, \quad (24)\end{aligned}$$

which shows that as the cut-off frequency is approached, the group velocity tends to zero. Below cut-off, however, the group delay is imaginary, which implies an infinite (real) group velocity. The group-velocity dispersion [45] (GVD) is given by

$$GVD = \partial^2\varphi(\omega)/\partial\omega^2 = -\frac{L^4}{c^4}\frac{\omega_c^2}{\varphi^3(\omega)}, \quad (25)$$

which is also imaginary below the cut-off frequency. It can be shown that around zero frequency, this GVD gives rise to a broadening of the pulse spectrum and a narrowing of the pulse width.It can be shown that the waveguide propagation equation given here is causal. The accumulated phase in an unended waveguide given by Eq. (22) corresponds to the dielectric function

$$\varepsilon(\omega) = (1 - \omega_c^2/\omega^2). \quad (26)$$

It can be seen that the dielectric function is real for all frequencies and becomes negative below the cut-off frequency. Using the Kramers–Kronig relations, one can calculate the imaginary part by contour integration (P is the principal part): [46]

$$\varepsilon'' = \frac{1}{\pi} P \int \frac{\omega_c^2/x^2}{x-\omega} \mathrm{d}x = \frac{\omega_c^2}{\pi} P \int \frac{1}{x^2(x-\omega)} \mathrm{d}x$$
$$= \frac{\omega_c^2}{\pi} 2\pi \mathrm{i} \left(\frac{1/2}{\omega^2} + \frac{-1/2}{(-\omega)^2} \right) = 0, \quad (27)$$

which proves that the dielectric function as given in Eq. (26) is causal.

The expressions for waveguide propagation given above do not include the effect of coupling the electromagnetic waves into and out of the waveguide. This approximation is only valid if the waveguide is long enough that one may ignore the effects of multiple reflections inside the waveguide. From the expression for the accumulated phase, Eq. (22), one can derive the effective index as $n(\omega) = c\varphi/\omega L$. If the waveguide is embedded in air, the interfaces at the waveguide entrance and exit will act as mirrors. Evanescent waves cannot interfere but each reflection off an interface results in a phase shift and a corresponding time shift. As these shifts accumulate on multiple reflections inside the waveguide, this may lead to a significant change in the properties of the transmitted field. Using Fresnel coefficients [59] and including the effects of multiple reflections, one can derive the frequency-dependent transmission function using the geometric series as

$$\tilde{T}(\omega) = \frac{\tilde{\alpha}(\omega)\mathrm{e}^{\mathrm{i}kL}}{1 - \tilde{\beta}(\omega)\mathrm{e}^{\mathrm{i}2kL}}, \quad (28)$$

where

$$\tilde{\alpha}(\omega) = \frac{4n(\omega)}{(1+n(\omega))^2}, \quad \tilde{\beta}(\omega) = \frac{(1-n(\omega))^2}{(1+n(\omega))^2}, \quad (29)$$

and $k = \omega n(\omega)/c$ is the wavenumber in the waveguide.

The time-response function is given by the Fourier transform of Eq. (8), which has infinitely many poles in the complex-frequency plane given by the equation $\tilde{\beta}(\omega)\exp(\mathrm{i}2kL) = 1$. In order to prove that the response is causal, one has to show that all poles are in the lower-half complex-frequency plane [46,49]. It appears that an analytical expression for the poles cannot be found. A numerical search (see Fig. 6) only yielded poles in the lower-half frequency plane.

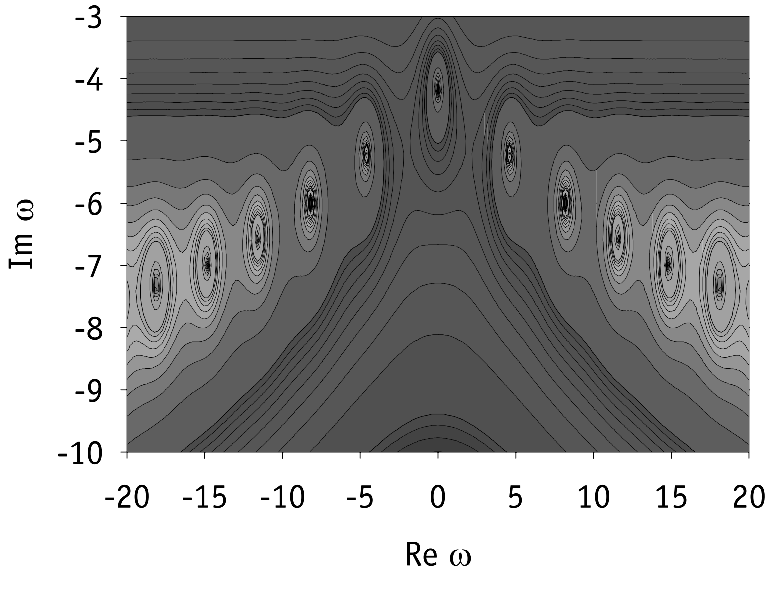

Fig. 6. Contour plot of $\log_{10}|T(\omega)|$ (Eq. 28) for $\omega_c = c = L = 1$ showing the beginning of an infinite series of poles in the lower-half complex-frequency plane.

The equations given above allow one to calculate the transmitted electromagnetic field with or without the effects of coupling the radiation in an out of the waveguide. In order to calculate the field distribution inside the waveguide, one has to solve the Maxwell wave equation directly. Consider an electromagnetic field impinging on a waveguide suspended in a vacuum. If this field is written as:

$$E(z,\omega) = \Psi(z,\omega)\exp(-\mathrm{i}\omega t), \tag{30}$$

the scalar component is governed by the Helmholtz wave equation [2]

$$\frac{\partial^2 \Psi(z,\omega)}{\partial z^2} = -\left(\frac{\omega n(\omega)}{c}\right)^2 \Psi(z,\omega) \equiv -k^2 \Psi(z,\omega). \tag{31}$$

For a cylindrical metal waveguide, the TE mode wavenumber is [58]

$$k = c^{-1}\sqrt{\omega^2 - \omega_c^2} \tag{32}$$

with the critical frequency given by Eq. (22). There are three regions to be considered (see Fig. 7), 1: in front of the waveguide, 2: inside the waveguide and 3: beyond the waveguide. The solutions to the wave equation can be written as:

$$\begin{cases} \Psi_1(z,\omega) = \mathrm{e}^{\mathrm{i}k_1 z} + R\mathrm{e}^{-\mathrm{i}k_1 z}, \\ \Psi_2(z,\omega) = A\mathrm{e}^{\mathrm{i}k_2 z} + B\mathrm{e}^{-\mathrm{i}k_2 z}, \\ \Psi_3(z,\omega) = T\mathrm{e}^{\mathrm{i}k_1 (z-d)}. \end{cases} \tag{33}$$

Note that the fields in the above equation are defined with the origin at the first interface. The phase factor in the third term ($\mathrm{e}^{-\mathrm{i}k_1 d}$) makes that propagation in the gap is included in the T term.

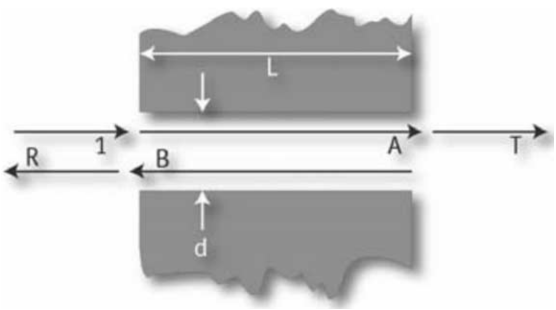

Fig. 7. Schematic diagram of a waveguide and the incoming and outgoing waves.

With the E-field s-polarised and assuming the magnetic permeability to be unity everywhere, both field and the spatial derivative of the field are continuous across the boundaries [2]. Therefore, it follows:

$$\begin{cases} \Psi_1(0,\omega) = \Psi_2(0,\omega), \Psi_2(d,\omega) = \Psi_3(d,\omega) \\ \Psi'_1(0,\omega) = \Psi'_2(0,\omega), \Psi'_2(d,\omega) = \Psi'_3(d,\omega) \end{cases} \tag{34}$$

Solving this set of linear equations in the parameters A, B, R, and T, it is found ($n = ck/\omega$):

$$T = \frac{\alpha_T \mathrm{e}^{\mathrm{i}dk_2}}{1-\beta\mathrm{e}^{2\mathrm{i}dk_2}}, \quad \alpha_T = \frac{4k_1k_2}{(k_1+k_2)^2}, \tag{35}$$

$$R = \frac{\alpha_R(1-\mathrm{e}^{2\mathrm{i}dk_2})}{1-\beta\mathrm{e}^{2\mathrm{i}dk_2}}, \quad \alpha_R = \frac{(k_1^2-k_2^2)}{(k_1+k_2)^2}, \tag{36}$$

$$A = \frac{\alpha_A}{1-\beta\mathrm{e}^{2\mathrm{i}dk_2}}, \quad \alpha_A = \frac{2k_1(k_1+k_2)}{(k_1+k_2)^2}, \tag{37}$$

$$B = \frac{\alpha_B \mathrm{e}^{2\mathrm{i}dk_2}}{1-\beta\mathrm{e}^{2\mathrm{i}dk_2}}, \quad \alpha_B = \frac{-2k_1(k_1-k_2)}{(k_1+k_2)^2}, \tag{38}$$

with

$$\beta = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}. \tag{39}$$

The result for the transmission and reflection is identical to that obtained using Fresnel coefficients (Eq. 28).

Consider a pulse in the time domain:

$$E(z,t) = F(\tau)\mathrm{e}^{-\mathrm{i}\omega_0\tau}, \quad \tau = t - zn/c. \tag{40}$$

Its Fourier transform is:

$$\int E(z,t)\mathrm{e}^{+\mathrm{i}\omega t}\,\mathrm{d}t = \tilde{F}(\omega-\omega_0)\mathrm{e}^{\mathrm{i}zn\omega/c} = \tilde{F}(\omega-\omega_0)\mathrm{e}^{\mathrm{i}kz}. \tag{41}$$

Therefore, to calculate what will happen to a pulse travelling through a barrier, one can take the solution Eq. (33) and multiply it with $\tilde{F}(\omega-\omega_0)\mathrm{e}^{-\mathrm{i}\omega t}$ and integrate (sum) over all frequency components of the pulse, i.e.:

$$E(z,t) = \frac{1}{2\pi}\int \tilde{F}(\omega-\omega_0)\Psi(z,\omega)\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega. \tag{42}$$

This method has been used to do all of the time-domain simulations in this paper, where the Fourier transforms were performed using FFT [52].

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